December 2022 update

See below the latest updates for the current 20th edition, which are all incorporated in the PDF file of the Kourovka Notebook available at `https://kourovka-notebook.org` (see also `https://arxiv.org/pdf/1401.0300.pdf`).

For convenience of the readers all the updates since the first appearance of the 20th edition are also listed below; the newest comments or solutions added in December 2022 are marked **NEW**.

12.48. Let G be a sharply doubly transitive permutation group on a set Ω (see Archive 11.52 for a definition).

(a) Does G possess a regular normal subgroup if a point stabilizer is locally finite?

(b) Does G possess a regular normal subgroup if a point stabilizer has an abelian subgroup of finite index?

Comments of 2022: an affirmative answer to part (b) is obtained in permutation characteristic 0 (F. O. Wagner, *Preprint*, 2022, `https://hal.archives-ouvertes.fr/hal-03590818`).

V. D. Mazurov

14.42. Is a free pro-p-group representable by matrices over an associative-commutative profinite ring with 1?

A negative answer is equivalent to the fact that every linear pro-p-group satisfies a non-trivial pro-p-identity; this is known to be true in dimension 2 (for $p \neq 2$: A. N. Zubkov, *Siberian Math. J.*, **28**, no. 5 (1987), 742–747; for $p = 2$: D. E.-C. Ben-
NEW Ezra, E. Zelmanov, *Trans. Amer. Math. Soc.*, **374**, no. 6 (2021), 4093–4128).

A. N. Zubkov, V. N. Remeslennikov

14.53. *Conjecture:* Let G be a profinite group such that the set of solutions of the equation $x^n = 1$ has positive Haar measure. Then G has an open subgroup H and an element t such that all elements of the coset tH have order dividing n.

This is true in the case $n = 2$. It would be interesting to see whether similar results hold for profinite groups in which the set of solutions of some equation has positive measure.

Comment of 2021: This is also proved for $n = 3$ (A. Abdollahi, M. S. Malekan,
NEW *Adv. Group Theory Appl.*, **13** (2022), 71–81).

L. Levai, L. Pyber

NEW *__17.42.__ Let $\overline{G}$ be a simple algebraic group of adjoint type over the algebraic closure $\overline{F}_p$ of a finite field F_p of prime order p, and σ a Frobenius map (that is, a surjective homomorphism such that $\overline{G}_\sigma = C_{\overline{G}}(\sigma)$ is finite). Then $G = O^{p'}(\overline{G}_\sigma)$ is a finite group of Lie type. For a maximal σ-stable torus $\overline{T}$ of $\overline{G}$, let $N = N_{\overline{G}}(\overline{T}) \cap G$. Assume also that G is simple and $G \ncong \mathrm{SL}_3(2)$. Does there always exist $x \in G$ such that $N \cap N^x$ is a p-group?

E. P. Vdovin

*A definitive answer for a stronger question on the size of a base has been obtained in (T. C. Burness, A. R. Thomas, *to appear in J. Algebra*, `https://arxiv.org/pdf/2207.09495.pdf`), which implies the following answer (in the notation therein): there is $x \in G$ such that $N \cap N^x$ is a p-group if and only if (G, N) is not one of the following: $(\mathrm{L}_3(2), 7{:}3)$, $(\mathrm{U}_4(2), 3^3{:}S_4)$, $(\mathrm{U}_5(2), 3^4{:}S_5)$.

***19.23.** For a group G, let $\mathrm{Tor}_1(G)$ be the normal closure of all torsion elements of G, and then by induction let $\mathrm{Tor}_{i+1}(G)$ be the inverse image of $\mathrm{Tor}_1(G/\mathrm{Tor}_i(G))$. The torsion length of G is defined to be either the least positive integer l such that $G/\mathrm{Tor}_l(G)$ is torsion-free, or ω if no such integer exists (since $G/\bigcup \mathrm{Tor}_i(G)$ is always torsion-free).

Does there exists a finitely generated, or even finitely presented, soluble group with torsion length greater than 2? *M. Chiodo, R. Vyas*

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*Yes, such groups exist (I. J. Leary, A. Minasyan, *to appear in J. Group Theory*, `https://arxiv.org/abs/2112.14546`).

***19.24.** For a group G, let $\mathrm{Tor}(G)$ be the normal closure of all torsion elements of G. Does there exist a finitely presented group G such that $G/\mathrm{Tor}(G)$ is not finitely presented? Such a group must necessarily be non-hyperbolic. *M. Chiodo, R. Vyas*

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*Yes, such groups exist: one soluble example and another virtually torsion-free are constructed in (I. J. Leary, A. Minasyan, *to appear in J. Group Theory*, `https://arxiv.org/abs/2112.14546`).

19.61. Let $\mathfrak{A} = \{\mathfrak{A}_r \mid r \in \Phi\}$ be an elementary carpet of type Φ over a commutative ring K (see 7.28), and let $\Phi(\mathfrak{A}) = \langle x_r(\mathfrak{A}_r) \mid r \in \Phi\rangle$ be its carpet subgroup. Define the *closure* of the carpet $\mathfrak{A}$ to be the set of additive subgroups $\overline{\mathfrak{A}} = \{\overline{\mathfrak{A}}_r \mid r \in \Phi\}$, where $\overline{\mathfrak{A}}_r = \{t \in K \mid x_r(t) \in \Phi(\mathfrak{A})\}$. Is the closure $\overline{\mathfrak{A}}$ of a carpet $\mathfrak{A}$ always a carpet?

NEW An affirmative answer is known if $\Phi = A_l, D_l, E_l$. *Ya. N. Nuzhin*

***20.27.** Let G be a finite group, p a prime number, and let $|x^G|_p$ denote the maximum power of p that divides the class size of an element $x \in G$. Suppose that there exists a p-element $g \in G$ such that $|g^G|_p = \max_{x\in G} |x^G|_p$. Is it true that G has a normal p-complement?

A partial answer is in (`https://arxiv.org/abs/1812.03641`). *I. B. Gorshkov*

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*No, not necessarily, a counterexample is given by `SmallGroup(192,945)` (B. Sambale, *Letter of 16 February 2022*).