June 2022 update

See below the latest updates for the current 20th edition, which are all incorporated in the PDF file of the Kourovka Notebook available at `https://kourovka-notebook.org` (see also `https://arxiv.org/pdf/1401.0300.pdf`).

For convenience of the readers all the updates since the first appearance of the 20th edition are also listed below; the newest comments or solutions added in June 2022 are marked **NEW**.

12.48. Let G be a sharply doubly transitive permutation group on a set Ω (see Archive 11.52 for a definition).

(a) Does G possess a regular normal subgroup if a point stabilizer is locally finite?

(b) Does G possess a regular normal subgroup if a point stabilizer has an abelian subgroup of finite index?

NEW *Comments of 2022*: an affirmative answer to part (b) is obtained in permutation characteristic 0 (F. O. Wagner, *Preprint*, 2022, `https://hal.archives-ouvertes.fr/hal-03590818`).

V. D. Mazurov

***19.23.** For a group G, let $\mathrm{Tor}_1(G)$ be the normal closure of all torsion elements of G, and then by induction let $\mathrm{Tor}_{i+1}(G)$ be the inverse image of $\mathrm{Tor}_1(G/\mathrm{Tor}_i(G))$. The torsion length of G is defined to be either the least positive integer l such that $G/\mathrm{Tor}_l(G)$ is torsion-free, or ω if no such integer exists (since $G/\bigcup \mathrm{Tor}_i(G)$ is always torsion-free).

Does there exists a finitely generated, or even finitely presented, soluble group with torsion length greater than 2?

M. Chiodo, R. Vyas

NEW *Yes, such groups exist (I. J. Leary, A. Minasyan, *to appear in J. Group Theory*, `https://arxiv.org/abs/2112.14546`).

***19.24.** For a group G, let $\mathrm{Tor}(G)$ be the normal closure of all torsion elements of G. Does there exist a finitely presented group G such that $G/\mathrm{Tor}(G)$ is not finitely presented? Such a group must necessarily be non-hyperbolic.

M. Chiodo, R. Vyas

NEW *Yes, such groups exist: one soluble example and another virtually torsion-free are constructed in (I. J. Leary, A. Minasyan, *to appear in J. Group Theory*, `https://arxiv.org/abs/2112.14546`).

NEW ***20.27.** Let G be a finite group, p a prime number, and let $|x^G|_p$ denote the maximum power of p that divides the class size of an element $x \in G$. Suppose that there exists a p-element $g \in G$ such that $|g^G|_p = \max_{x\in G} |x^G|_p$. Is it true that G has a normal p-complement?

A partial answer is in (`https://arxiv.org/abs/1812.03641`).

I. B. Gorshkov

*No, not necessarily, a counterexample is given by `SmallGroup(192,945)` (B. Sambale, *Letter of 16 February 2022*).